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Formal Definitions and Closure Conditions of the χ_{MDR}-Lagrange Structure

This appendix establishes all formal definitions, normalizations, and model limits of the ISOCH action framework. It serves to uniquely identify all quantities used in the text, their dynamics, coupling conditions, and normalized boundary values.

Empirical Derivation and Theoretical Embedding of the χ_{MDR} Drift

The trend of the matter-dynamics rate χ_{MDR} is derived exclusively from mutually independent observational data sets. In the CMB phase-shift test (Planck 2018 data), the first empirical fixed-point determination of the χ_{MDR} drift is obtained from the phase position of the acoustic peaks $n = 1–6$.

In a second, fully independent Lyman-α-BAO test (BOSS / eBOSS, $z = 2.3– 2.55$), the same drift behavior is found. Both data sets are physically independent and define the observationally based form of the epoch-dependent dynamics rate.

This empirically determined trend is not used as input to the Lagrangian variation, but exclusively for the calibration of the free parameters of the ISOCH potential $V(\chi_{\mathrm{MDR}}, \varepsilon)$.

On this basis, all theoretical quantities become predictive, including the epoch-dependent energy density $\Omega_\chi(\varepsilon)$, the expansion history $H(\varepsilon)$, the local value of H_0, and residual effects in CMB and BAO measurements.

For every computation, every fit relation, and every derived formula, the following scheme is applied strictly once:

empirical χ_{MDR} drift → single potential calibration → theoretical prediction.

The multi-line test verifies the internal consistency of this method by demonstrating that different spectral lines and transitions within the same epoch window yield the same $\chi_{\mathrm{EPO}}(\varepsilon)$ normalization profile. The validation test then confirms that the once-calibrated potential $V(\chi_{\mathrm{MDR}}, \varepsilon)$ reproduces all observed epoch relations without any further adjustment.

This excludes any multiple use of empirical relations or their appearance as ISOCH “predictions”. The entire variational system is formally closed, empirically consistent, and logically non-circular.

Geometric Framework

The geometry follows the FLRW metric $g_{\mu\nu} = \mathrm{diag}(-, +, +, +)$ and is not varied.

$$\delta S_{\mathrm{geo}} = 0.$$

An Einstein–Hilbert term is not used. The geometric background quantity $H(t) = \dot{a}/a$ is empirically determined and not a variational object. The Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho_m$$

serves as the reference equation of GR, not as a consequence of the ISOCH action.

Dynamic and Non-Dynamic Quantities

Symbol	Bedeutung	Dynamisch
χ_{MDR}	Matter-dynamics rate (process-normalized scalar variable)	✅
$K_{\chi_{\mathrm{MDR}}}$	kinetic normalization factor, positive, normalized to H_0	❌
$V(\chi_{\mathrm{MDR}}, \varepsilon)$	epoch-dependent potential	❌
ε	epoch-coordinated normalization $\varepsilon = \ln a = -\ln(1+z)$	❌
$H(t)$	empirical function of the FLRW background	❌
$\mathcal{M}(\chi_{\mathrm{em}}, \chi_{\mathrm{obs}})$	observer–emitter normalization function	❌
ρ_c	critical energy density $= 3H_0^2/(8\pi G)$	❌

$$\frac{\partial V}{\partial \varepsilon} = 0, \qquad \nabla_\mu T^{\mu\nu} = 0.$$

Lagrangian Density and Variational Principle

$$\mathcal{L}_{\chi_{\mathrm{MDR}}} = \frac{1}{2} K_{\chi_{\mathrm{MDR}}} g^{\mu\nu} \nabla_\mu \chi_{\mathrm{MDR}} \nabla_\nu \chi_{\mathrm{MDR}} - V(\chi_{\mathrm{MDR}}, \varepsilon).$$

The variation with respect to χ_{MDR} yields:

$$K_{\chi_{\mathrm{MDR}}} \nabla_\mu \nabla^\mu \chi_{\mathrm{MDR}} + \frac{\partial V}{\partial \chi_{\mathrm{MDR}}} = 0.$$

The geometry is assumed to be observation-fixed; no variation with respect to $g_{\mu\nu}$ takes place. Thus, ISOCH describes matter dynamics within the empirically defined FLRW background, where $H(\varepsilon)$ is treated as a fixed background quantity. This approach ensures that the Lagrangian formalism acts exclusively on the matter component and avoids a double variation of the spacetime metric.

In FLRW:

$$\ddot{\chi}_{\mathrm{MDR}} + 3H\dot{\chi}_{\mathrm{MDR}} + \frac{1}{K_{\chi_{\mathrm{MDR}}}} \frac{\partial V}{\partial \chi_{\mathrm{MDR}}} = 0.$$

Energy–Momentum Tensor and Continuity

$$T_{\mu\nu} = K_{\chi_{\mathrm{MDR}}}\, \partial_\mu \chi_{\mathrm{MDR}}\, \partial_\nu \chi_{\mathrm{MDR}} - g_{\mu\nu} \left[\frac{1}{2} K_{\chi_{\mathrm{MDR}}}\, \partial_\alpha \chi_{\mathrm{MDR}}\, \partial^\alpha \chi_{\mathrm{MDR}} - V(\chi_{\mathrm{MDR}}, \varepsilon)\right].$$

$$\rho_{\chi_{\mathrm{MDR}}} = \frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2 + V, \qquad p_{\chi_{\mathrm{MDR}}} = \frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2 - V.$$

$$\dot{\rho}_{\chi_{\mathrm{MDR}}} + 3H\big(\rho_{\chi_{\mathrm{MDR}}} + p_{\chi_{\mathrm{MDR}}}\big) = 0.$$

The condition $\nabla_\mu T^{\mu\nu} = 0$ is satisfied. $T_{\mu\nu}$ is defined as the functional derivative of the matter part of the action, while the geometry remains externally fixed. The background fulfills $\delta S_{\mathrm{geo}} = 0$ and $\nabla_\mu T^{\mu\nu} = 0$ follows as a subsidiary condition. Thus, the definition of the energy–momentum tensor remains fully consistent with the fixed FLRW geometry.

Potential Forms and Normalization

Linear Potential:

$$V_{\mathrm{lin}}(\chi_{\mathrm{MDR}}) = \Lambda^3_{\chi_{\mathrm{MDR}}} (\chi_{\mathrm{MDR}} - 1), \qquad V_{\mathrm{lin}}(1) = 0.$$

For the linear potential, the domain of definition is $\chi_{\mathrm{MDR}} \geq 1$; thus $V_{\mathrm{lin}} \geq 0$, and the WEC condition is globally satisfied. This restriction defines the admissible range of the matter-dynamics rate and excludes negative energy densities.

Quadratic Potential (Standard Case):

$$V_{\mathrm{quad}}(\chi_{\mathrm{MDR}}) = \frac{1}{2} m^2_{\chi_{\mathrm{MDR}}} (\chi_{\mathrm{MDR}} - 1)^2, \qquad V_{\mathrm{quad}}(1) = 0.$$

The quadratic potential is the primary model case for all variational and dynamic derivations in ISOCH. It possesses a well-defined minimum at $\chi_{\mathrm{MDR}} = 1$, and $m^2_{\mathrm{eff}} > 0$ ensures the stability of the χ_{MDR}-Dynamik. The linear potential is not used as an independent variational potential but solely as a local effective approximation for $|\chi_{\mathrm{MDR}} - 1| \ll 1$ and for residual or sensitivity comparisons.

The epoch dependence of the potential appears exclusively through empirically calibrated parameters $\alpha(\varepsilon)$; explicit variation with respect to ε is omitted. Formally, therefore,

$$V\big(\chi_{\mathrm{MDR}}; \alpha(\varepsilon)\big) \ \text{ with } \ \partial\varepsilon V = 0.$$

This representation ensures the consistency of the variational principle without additional source terms.

Epoch Function and Redshift Relation

This relation serves solely to connect theoretical and observed quantities. It is not a result of the variation of the action but is defined as a measurement postulate. $\mathcal{M}$ thus functions as a bridge between model parameters and observational data.

$$\varepsilon = f(z) \equiv \ln a\,(z) = -\ln(1 + z).$$

$$1 + z = \mathcal{M}(\chi_{\text{em}}, \chi_{\text{obs}}), \qquad \mathcal{M}(\chi_{\text{em}}, \chi_{\text{obs}}) = \frac{\chi_{\text{obs}}}{\chi_{\text{em}}}.$$

For $\chi_{\text{MDR}} \to 1$, it follows that $\mathcal{M} \to 1$ and $z \to 0$.

Linear Perturbations

$$\delta\ddot{\chi}_{\text{MDR}} + 3H\delta\dot{\chi}_{\text{MDR}} + \left(\frac{k^2}{a^2} + m_{\text{eff}}^2\right)\delta\chi_{\text{MDR}} = 0, \qquad m_{\text{eff}}^2 = V''|_{\chi_{\text{MDR}}}.$$

Solution in the Underdamped Regime:

$$\delta\chi_{\text{MDR}} \propto e^{-3Ht/2}\sin(\omega_k t + \phi), \qquad \omega_k^2 > \left(\frac{3H}{2}\right)^2.$$

Friedmann Closure

Since the geometry is not varied, the closure is carried out empirically via:

$$H(\varepsilon) = H_0\sqrt{\Omega_m e^{-3\varepsilon} + \Omega_{\chi_{\text{MDR}}}(\varepsilon)}.$$

$\Omega_{\chi_{\text{MDR}}}(\varepsilon)$ is defined from the ISOCH energy density:

$$\Omega_{\chi_{\text{MDR}}}(\varepsilon) = \frac{8\pi G}{3H_0^2}\left[\frac{1}{2}K_{\chi_{\text{MDR}}}(\dot{\chi}_{\text{MDR}})^2 + V(\chi_{\text{MDR}}, \varepsilon)\right].$$

With the calibrated parameters

$$\frac{\Lambda_{\chi_{\text{MDR}}}^3}{K_{\chi_{\text{MDR}}}} \sim H_0^3, \text{ und } \frac{m_{\chi_{\text{MDR}}}^2}{K_{\chi_{\text{MDR}}}} \sim H_0^2.$$

the value for the present epoch $\varepsilon = 0$ is obtained as $\Omega_{\chi_{\text{MDR}}}(0) \approx 0.73$. This confirms the internal consistency of the normalization and allows direct comparability with the observed density parameters.

GR Limit

$$\chi_{\text{MDR}} \to 1 \quad \Rightarrow \quad \dot{\chi}_{\text{MDR}} \to 0,\ V(\chi_{\text{MDR}}, \varepsilon) \to 0, \quad \Rightarrow \quad \rho_{\chi_{\text{MDR}}}, p_{\chi_{\text{MDR}}} \to 0,$$

$$H^2 \to \frac{8\pi G}{3}\rho_m.$$

Normalization Factor and Dimensions

For unambiguous reproducibility, $K_{\chi_{\mathrm{MDR}}}$ is fixed to 1 in units of H_0^2. All specified parameters and sensitivities refer to this absolute normalization.

$$K_{\chi_{\mathrm{MDR}}} = K_0 > 0, \qquad K_0 \equiv 1 \text{ in units of } H_0^2.$$

$$[\chi_{\mathrm{MDR}}] = 1, \quad \left[K_{\chi_{\mathrm{MDR}}}\right] = 1, \quad [V] = [H^2], \quad [\rho] = [H^2].$$

Parametric Calibration

$$\frac{\Lambda^3_{\chi_{\mathrm{MDR}}}}{K_{\chi_{\mathrm{MDR}}}} \sim H_0^3, \qquad \frac{m^2_{\chi_{\mathrm{MDR}}}}{K_{\chi_{\mathrm{MDR}}}} \sim H_0^2.$$

The parameters are determined by minimization over the observed χ_{MDR} fluxes: This empirical fit serves exclusively to determine the numerical parameter combinations $\Lambda^3_{\chi_{\mathrm{MDR}}}/K_{\chi_{\mathrm{MDR}}}$ und $m^2_{\chi_{\mathrm{MDR}}}/K_{\chi_{\mathrm{MDR}}}$ externally; it does not feed back into the Lagrangian variation and does not modify the action or the equation-of-motion system of the χ_{MDR} dynamics.

$$\min_{\mathrm{param}} \sum_i \frac{\left(\chi_{\mathrm{obs},i} - \chi_{\mathrm{MDR,model},i}\right)^2}{\sigma_i^2}$$

Stability Conditions

$$K_{\chi_{\mathrm{MDR}}} > 0, \qquad m^2_{\mathrm{eff}} > 0.$$

Thus, no tachyonic or ghost modes exist.

Energy Conservation, WEC Condition, and Closure Relations

The process-normalized matter-dynamics rate χ_{MDR} satisfies the weak energy condition (WEC) and local energy conservation. With positive, constant $K_{\chi_{\mathrm{MDR}}}$ and epoch-fixed $H(\varepsilon)$, the following holds:

$$\rho_{\chi_{\mathrm{MDR}}} + p_{\chi_{\mathrm{MDR}}} = K_{\chi_{\mathrm{MDR}}}(\dot{\chi}_{\mathrm{MDR}})^2 \geq 0, \quad \rho_{\chi_{\mathrm{MDR}}} \geq 0, \quad V(\chi_{\mathrm{MDR}}, \varepsilon) \geq 0.$$

Local energy conservation follows directly from $\nabla_\mu T^{\mu\nu} = 0$:

$$\dot{\rho}_{\chi_{\mathrm{MDR}}} + 3H(\varepsilon)\left(\rho_{\chi_{\mathrm{MDR}}} + p_{\chi_{\mathrm{MDR}}}\right) = 0.$$

Thus, the weak energy condition is globally satisfied, and the dynamics are energetically and consistently closed.

For completeness of the model closure, the following also apply:

1. Geometry fixed (no Einstein–Hilbert term).
2. ε non-variable ($\partial V / \partial \varepsilon = 0$).

3. H empirically defined $\big(H = H(\varepsilon)\big)$.
4. Potentials normalized $(V(1) = 0)$.
5. Linear perturbations in the underdamped regime.
6. $K_{\chi_{\mathrm{MDR}}}$ constant and positive.
7. Calibration via observed χ_{MDR} fluxes.
8. Energy conservation ensured.
9. GR limit reproduced.

Final Equation of Model Closure

The ISOCH theory is closed by:

$$\big(\chi_{\mathrm{MDR}}, \dot{\chi}_{\mathrm{MDR}}, V, K_{\chi_{\mathrm{MDR}}}, H, \varepsilon\big)$$

with the following conditions:

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \partial V/\partial \varepsilon = 0, \qquad K_{\chi_{\mathrm{MDR}}} = \mathrm{const.}, H = H(\varepsilon).$$

It contains no undefined quantities and no external variational parameters.

Supplementary Section – Clarifications on Variational Structure, Domain of Validity, and Empirical Coupling

This section resolves remaining definitional ambiguities in the formulation of the potential, the variational domain, the energy conditions, and the empirical H- closure. All clarifications serve to ensure the complete self-consistency of the ISOCH action framework.

Potential Structure and ε- Dependence

The potential is defined exclusively as a function of χ_{MDR}:

$$V = V(\chi_{\mathrm{MDR}}),$$

with ε serving as a non-variable epochal normalization. An ε -dependence can only be introduced indirectly through the calibration parameter fit, for example via empirical functions $V\big(\chi_{\mathrm{MDR}};\ \alpha(\varepsilon)\big)$. Accordingly, the consistent condition holds:

$$\frac{\partial V}{\partial \varepsilon} = 0, \qquad \frac{\mathrm{d}V}{\mathrm{d}\varepsilon} = \frac{\partial V}{\partial \chi_{\mathrm{MDR}}} \frac{\mathrm{d}\chi_{\mathrm{MDR}}}{\mathrm{d}\varepsilon}.$$

This representation preserves the variational condition and fully resolves the apparent contradiction between “epoch-dependent” and “non-variable.

WEC Domain and Linear Potential

For the linear potential, the following holds:

$$V_{\mathrm{lin}}(\chi_{\mathrm{MDR}}) = \Lambda^3_{\chi_{\mathrm{MDR}}}(\chi_{\mathrm{MDR}} - 1), \qquad V_{\mathrm{lin}}(1) = 0.$$

There exists an explicit domain of definition

$$\chi_{\mathrm{MDR}} \geq 1,$$

which ensures $V_{\mathrm{lin}} \geq 0$ and that the weak energy condition (WEC) is globally satisfied:

$$\rho_{\chi_{\mathrm{MDR}}} + p_{\chi_{\mathrm{MDR}}} = K_{\chi_{\mathrm{MDR}}}(\dot{\chi}_{\mathrm{MDR}})^2 \geq 0.$$

For global normalization, the offset-normalized form can equivalently be used:

$$V_{\mathrm{lin,norm}}(\chi_{\mathrm{MDR}}) = \Lambda^3_{\chi_{\mathrm{MDR}}}\,|\chi_{\mathrm{MDR}} - 1|.$$

This representation preserves $V \geq 0$ over the entire domain of definition without altering the behavior of the equation of motion or the stability conditions. Thus, the WEC domain is uniquely determined, and the energy positivity of the ISOCH potential is globally ensured.

The validity range of the linear potential

$$V_{\mathrm{lin}}(\chi_{\mathrm{MDR}}) = \Lambda^3_{\chi_{\mathrm{MDR}}}(\chi_{\mathrm{MDR}} - 1)$$

is restricted to the domain $\chi_{\mathrm{MDR}} \geq 1$. It is used exclusively as an effective approximation in the immediate vicinity of $\chi_{\mathrm{MDR}} \approx 1$. The standard evolution and all variational proofs of the ISOCH model are based on the quadratic potential V_{quad}.

Empirical H-Closure and Algorithmic Corollary

The Hubble function $H(\varepsilon)$ is not a variational object but an empirically defined input term of the background. It is not derived from the variation of the action but is set based on observational constraints and serves as a constraint for fixing the geometry.

$$H(\varepsilon) = H_0\sqrt{\Omega_m e^{-3\varepsilon} + \Omega_{\chi_{\mathrm{MDR}}}(\varepsilon)}.$$

The ISOCH energy density of the matter-dynamics rate is given by:

$$\Omega_{\chi_{\mathrm{MDR}}}(\varepsilon) = \frac{8\pi G}{3H_0^2}\left[\frac{1}{2}K_{\chi_{\mathrm{MDR}}}(\dot{\chi}_{\mathrm{MDR}})^2 + V(\chi_{\mathrm{MDR}})\right].$$

For numerical determination, $H(\varepsilon)$ is computed iteratively. Starting from $H^{(0)} = H_0$, the following applies:

$$H^{(n+1)} = H_0\sqrt{\Omega_m e^{-3\varepsilon} + \Omega^{(n)}_{\chi_{\mathrm{MDR}}}}\,;$$

The computational step proceeds algorithmically:

compute $\Omega^{(n)}_{\chi_{\mathrm{MDR}}}$ from $\chi^{(n)}_{\mathrm{MDR}}$; update $H^{(n+1)}$ according to the above relation.

The relation is contractive in the range $H/H_0 \in [0.5, 1.5]$. Since $\frac{\partial H^{(n+1)}}{\partial H^{(n)}} < 1$ is satisfied for all physically relevant parameters, the sequence $H^{(n)}$ converges to a unique fixed point $H(\varepsilon)$. Thus, the existence and uniqueness of the empirical closure are formally guaranteed.

For formal clarification of the empirical–variational separation, the following applies:

The ISOCH theory treats the Hubble function $H(\varepsilon)$ in two logically separated stages to ensure self-consistency between the empirically calibrated geometry and the energy density $\Omega_{\chi_{\mathrm{MDR}}}(\varepsilon)$ defined by χ_{MDR}.

In the first stage, $H(\varepsilon)$ is treated as an observation-based fixed background quantity. During variation,

$$\delta g_{\mu\nu} = 0, \qquad \delta H(\varepsilon) = 0,$$

so that the action variation operates exclusively within the domain of matter dynamics χ_{MDR}. $H(\varepsilon)$ acts only as an exogenous damping term in the equations of motion.

In the second stage, after the variation, it is verified whether the energy density $\Omega_{\chi_{\mathrm{MDR}}}(\varepsilon)$ yields a closure relation

$$H^2(\varepsilon) = H_0^2\left[\Omega_m\, \mathrm{e}^{-3\varepsilon} + \Omega_{\chi_{\mathrm{MDR}}}(\varepsilon)\right]$$

consistent with the empirical observations of $H(\varepsilon)$. The fixed-point iteration thereby constitutes no additional variation but a subsequent consistency check between theoretical energy density and empirical geometry.

This formulation removes any apparent tension between the “exogenous” and “endogenous” treatments of $H(\varepsilon)$; the variation remains strictly confined to χ_{MDR}, while the geometry retains its empirically fixed character within the ISOCH theory.

The separation between the dynamic matter-dynamics rate χ_{MDR} and the fixed geometry $H(\varepsilon)$ remains unambiguous. The fixed-point mapping ensures a stable and deterministic closure of the ISOCH system.

Convergence of the H-Closure:

The iterative relation constitutes a contractive mapping in the range $H/H_0 \in [0.5, 1.5]$. Since $\partial H^{(n+1)} / \partial H^{(n)} < 1$ is satisfied for all physically relevant parameters, the sequence $H^{(n)}$ converges toward a unique fixed point $H(\varepsilon)$. Thus, the existence and uniqueness of the closure are mathematically guaranteed, even though H is not derived directly from variation but determined from observational constraints; the resulting fixed-point mapping nonetheless ensures a unique, convergent, and stable closure within the ISOCH regime.

This empirical closure does not couple the geometry dynamically to the action but serves as an observational constraint. Derivation of the Friedmann formula from the ISOCH action is not required; H represents a fixed background quantity. Thus, the separation between the dynamic matter rate χ_{MDR} and the fixed geometry $H(\varepsilon)$ is unambiguous.

Initial Conditions of the χ_{MDR} Dynamics

For the integration of the equation of motion, defined initial values are set:

$$\chi_{\mathrm{MDR}}(\varepsilon = 0) = 1, \qquad \dot{\chi}_{\mathrm{MDR}}(\varepsilon = 0) = 0.$$

These initial conditions fix the normalization of the matter-dynamics rate χ_{MDR} to the present epoch $\varepsilon = 0$ and ensure that $V(1) = 0$ and $\rho_{\chi_{\mathrm{MDR}}}, p_{\chi_{\mathrm{MDR}}} \to 0$ in the GR limit.

Numerical Calibration and Reproducibility

The parameter combinations

$$\frac{\Lambda^3_{\chi_{\mathrm{MDR}}}}{K_{\chi_{\mathrm{MDR}}}} \sim H_0^3, \qquad \frac{m^2_{\chi_{\mathrm{MDR}}}}{K_{\chi_{\mathrm{MDR}}}} \sim H_0^2.$$

are calibrated through empirical fits to Ω_{m} and $\Omega\chi_{\mathrm{MDR}}(\varepsilon = 0) \approx 0.73$. The calibration is performed by minimizing

$$\min_{\text{param}} \sum_i \frac{\left(\chi_{\mathrm{obs},i} - \chi_{\mathrm{MDR,model},i}\right)^2}{\sigma_i^2}$$

where H_0 and Ω_{m} are fixed, and χ_{MDR} is adjusted parametrically. This definition makes the ISOCH parameters fully reproducible.

Redshift Relation

The defined observer–emitter relation

$$1 + z = \mathcal{M}(\chi_{\mathrm{em}}, \chi_{\mathrm{obs}}) = \frac{\chi_{\mathrm{obs}}}{\chi_{\mathrm{em}}}$$

is postulated by the ISOCH normalization. It is not derived from the action but constitutes the epoch-dependent correspondence $\varepsilon \leftrightarrow z$. This uniquely establishes the observational normalization without introducing additional degrees of freedom.

Potential Definition (Consistency Theorem)

$$\text{We use } V = V(\chi_{\mathrm{MDR}}) \text{ with } \frac{\partial V}{\partial \varepsilon} = 0.$$

Epoch information enters exclusively through the parameterization $\alpha(\varepsilon)$ or via the fit $V\big(\chi_{\mathrm{MDR}}; \alpha(\varepsilon)\big)$, not through an explicit ε-variation. Thus, $\nabla_\mu T^{\mu\nu} = 0$ always holds, and no additional source-term coupling arises.

Variable Conventions (Dual Epoch Notation)

The theoretical formulation uses $\varepsilon = \ln a = -\ln(1 + z)$ as the internal coordinate and z as the empirical observational variable, connected by the bridging function

$$\varepsilon = f(z) = -\ln(1+z).$$

Thus, the notation is doubly defined but unambiguously linked; confusion between theoretical and observational normalization is excluded.

Fixed-Point Clarification "H Exogenous"

$H(\varepsilon)$ is not a variational object but an exogenous input term of the background. All expressions in the action use $H(\varepsilon)$ as a predetermined function; consequently, the Lagrangian variation is restricted solely to χ_{MDR}. This corresponds to the ISOCH definition of a fixed geometry without an Einstein–Hilbert term.

Convergence and Uniqueness of the H-Closure

The iterative relation

$$H^{(n+1)} = H_0\sqrt{\Omega_m e^{-3\varepsilon} + \Omega^{(n)}_{\chi_{\mathrm{MDR}}}(\varepsilon)}$$

constitutes a contractive mapping in the interval $H/H_0 \in [0.5,1.5]$. Since $\partial H^{(n+1)}/\,\partial H^{(n)} < 1$ holds for all physically relevant parameters, the sequence $H^{(n)}$ converges to a unique fixed point $H(\varepsilon)$. Thus, the existence and uniqueness of the closure are formally guaranteed.

Formal Convergence of the Empirical H-Closure

To ensure mathematical completeness, the fixed-point iteration used in the ISOCH formulation,

$$H_{n+1} = H_0\sqrt{\Omega_m e^{-3\varepsilon} + \Omega_{\chi_{\mathrm{MDR}}}(H_n)}$$

is formally demonstrated to be a contractive mapping.

The energy-density component is given by

$$\Omega_{\chi_{\mathrm{MDR}}}(H) = \frac{8\pi G}{3H_0^2}\left(\frac{1}{2}K_{\chi_{\mathrm{MDR}}}H^2\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2 + V(\chi_{\mathrm{MDR}})\right), \quad d\varepsilon = H\,dt.$$

Differentiation yields

$$F'(H) = \frac{H_0^2}{\sqrt{\Omega_m e^{-3\varepsilon} + \Omega_{\chi_{\mathrm{MDR}}}(H)}}\frac{d\Omega_{\chi_{\mathrm{MDR}}}}{dH}.$$

For the explicit part, the following holds

$$\left|\frac{d\Omega_{\chi_{\mathrm{MDR}}}}{dH}\right| \le A\,K_{\chi_{\mathrm{MDR}}}\,|H|\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2, \quad A = \frac{8\pi G}{3H_0^2}.$$

At the fixed point $H = H^*$, it follows

$$|F'(H^*)| \leq \frac{4\pi G}{3}\, K_{\chi_{\mathrm{MDR}}} \left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2.$$

With the empirically calibrated values

$$\left|\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right| \lesssim 0.04, \qquad K_{\chi_{\mathrm{MDR}}} = 1, \qquad \frac{8\pi G}{3} = 1,$$

one obtains

$$|F'(H^*)| \lesssim 8 \times 10^{-4} \ll 1.$$

According to the Banach fixed-point theorem, the mapping is strictly contractive; the iteration converges uniquely and independently of the initial value toward the fixed point $H(\varepsilon)$. Thus, the existence and uniqueness of the empirical H- closure are mathematically ensured.

Note: Relation (1) simultaneously defines a Lipschitz constant

$$L \leq 8 \times 10^{-4},$$

and all numerical integrations satisfy $L < 1$ in the range

$$H/H_0 \in [0.5,\ 1.5].$$

Hence, the numerical convergence of the empirically calibrated iteration is fully characterized. The subsequent derivation shows that this stability results from the general regularity conditions of the potential.

The ISOCH theory is therefore not only physically but also formally and analytically fully closed.

Lemma 1 – Contractive Property of the H-Iteration (Formal Proof)

For the ISOCH closure scheme

$$H_{n+1} = F(H_n) = H_0 \sqrt{\Omega_m e^{-3\varepsilon} + \Omega_{\chi_{\mathrm{MDR}}}(H_n)}$$

the following assumptions hold:

1. The potential $V(\chi_{\mathrm{MDR}})$ is continuously differentiable and possesses a finite Lipschitz constant $L_V > 0$:
$$\left|V'(\chi_{\mathrm{MDR},1}) - V'(\chi_{\mathrm{MDR},2})\right| \leq L_V \left|\chi_{\mathrm{MDR},1} - \chi_{\mathrm{MDR},2}\right|.$$
2. The effective mass parameter lies above a positive lower bound $m_{\mathrm{eff}}^2 \geq m_0^2 > 0$.
3. The deviation $|\chi_{\mathrm{MDR}} - 1|$ remains within a small, bounded region with $\delta \ll 1$.
4. The ratio H/H_0 lies within the empirically relevant interval $[0.5,1.5]$.

It then follows for the derivative of the iteration operator that:

$$|F'(H)| \leq C\, L_V\, \delta < 1,$$

where C is a dimensionless constant of order unity, resulting from the normalization of the energy and damping terms. Hence, F acts contractively on the interval $[0.5,1.5]$, and there exists exactly one stable fixed point

$$H^* = F(H^*).$$

This fixed point defines the unique self-consistency of the ISOCH closure for all potential forms V, that satisfy the stated regularity conditions.

The subsequent empirical demonstration confirms the numerical realization of the contraction theorem formally established here.

Extended Stability and Convergence Assurance under Implicit $\chi_{\mathrm{MDR}}(H)$ Coupling

The practical iteration employs the fixed-point mapping

$$H_{n+1} = H_0 \sqrt{\Omega_m e^{-3\varepsilon} + \Omega_{\chi_{\mathrm{MDR}}}(H_n)}.$$

Up to this point, the convergence proof has been conducted under the assumption that χ_{MDR} is already evaluated at each iteration step. To ensure stability also in the implicitly coupled case $\chi_{\mathrm{MDR}} = \chi_{\mathrm{MDR}}(H, \varepsilon)$—that is, during simultaneous determination of χ_{MDR} and H within a coupled integration routine—the contribution of the implicit derivative $\partial\chi'_{\mathrm{MDR}}/\,\partial H$ is taken into account.

With

$$\Omega_{\chi_{\mathrm{MDR}}}(H) = \frac{8\pi G}{3H_0^2}\left(\frac{1}{2}K_{\chi_{\mathrm{MDR}}}H^2\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2 + V(\chi_{\mathrm{MDR}})\right), \quad d\varepsilon = H\,dt.$$

the total derivative yields

$$\frac{d\Omega_{\chi_{\mathrm{MDR}}}}{dH} = \frac{\partial\Omega_{\chi_{\mathrm{MDR}}}}{\partial H} + \frac{\partial\Omega_{\chi_{\mathrm{MDR}}}}{\partial\chi'_{\mathrm{MDR}}}\frac{\partial\chi'_{\mathrm{MDR}}}{\partial H}.$$

The explicit term provides, as in the basic proof, the bound

$$|F'(H)| \leq L_0 = \frac{4\pi G}{3}\, K_{\chi_{\mathrm{MDR}}}\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2 \ll 1.$$

The backreaction of the χ_{MDR} dynamics on the H-iteration follows from the homogeneous equation of motion

$$K_{\chi_{\mathrm{MDR}}}\left(\chi''_{\mathrm{MDR}} + 3\chi'_{\mathrm{MDR}} + H^{-2}\,\frac{\partial V}{\partial\chi_{\mathrm{MDR}}}\right) = 0,$$

whose partial differentiation with respect to

$$K_{\chi_{\mathrm{MDR}}}\left(\frac{\partial\chi''_{\mathrm{MDR}}}{\partial H} + 3\,\frac{\partial\chi'_{\mathrm{MDR}}}{\partial H}\right) + \frac{\partial}{\partial H}\left(H^{-2}\,\frac{\partial V}{\partial\chi_{\mathrm{MDR}}}\right) = 0.$$

For $u(\varepsilon) = \partial\chi'_{\mathrm{MDR}} / \partial H$, a Grönwall estimate gives

$$\left|\frac{\partial\chi'_{\mathrm{MDR}}}{\partial H}\right| \leq C_\chi\, e^{-\int 3\, d\varepsilon} \leq C_\chi\, e^{-3\varepsilon},$$

where $C\chi$ is finite (e.g., from $\chi'_{\mathrm{MDR}}(0) = 0$. Thus, | $|\partial\chi'_{\mathrm{MDR}} / \partial H|$ remains bounded for all ε and decreases monotonically.

Substituting (Energy–Momentum Tensor) into (Euler–Lagrange equation) yields an effective Lipschitz constant

$$L_{\mathrm{eff}} = |F'(H)| \leq L_0 + L_\chi, \qquad L_\chi = \frac{4\pi G}{3}\, K_{\chi_{\mathrm{MDR}}} \left|\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right| \left|\frac{\partial\chi'_{\mathrm{MDR}}}{\partial H}\right|. \tag{5}$$

With $|d\chi_{\mathrm{MDR}}/d\varepsilon| \lesssim 0.04$ and $|\partial\chi'_{\mathrm{MDR}} / \partial H| \lesssim 10^{-2}$, it follows that $L_\chi \lesssim 10^{-6}$, and thus

$$L_{\mathrm{eff}} \approx L_0(1 + 10^{-3}) \ll 1.$$

The contraction condition $|F'(H)| < 1$ therefore also holds for the implicit coupling $\chi_{\mathrm{MDR}}(H, \varepsilon)$. The iterative H-closure thus converges uniquely and stably in both the explicit and implicitly coupled cases,

$$L_{\mathrm{eff}} < 1 \quad \forall\, \varepsilon \in R.$$

Hence, the stability of the ISOCH fixed-point procedure is formally proven even for simultaneously solved $\chi_{\mathrm{MDR}} - H$ couplings; the fixed-point procedure remains contractive, unique, and mathematically closed.

Variational Framework and Model Completeness

The ISOCH theory is defined as a partially variational theory. The variation of the action is performed exclusively within the space of the dynamic matter variable χ_{MDR}, while the geometric background function $H(\varepsilon)$ and the metric $g_{\mu\nu}$ are treated as empirically calibrated external constraints.

Thus, the framework does not represent an incomplete variational approach but a self-contained variational system within the χ_{MDR} subspace. The fixation of geometry corresponds to a fixed-background formalism: the spacetime metric is not varied but constitutes an observationally defined boundary condition of the partially variational action framework.

This procedure replaces spacetime variation with an internal process variation of matter dynamics $\chi_{\mathrm{MDR}}(\varepsilon)$ and leads to a mathematically closed set of independent Euler–Lagrange equations.

Formally, $\delta g_{\mu\nu} = 0$ and $\delta H(\varepsilon) = 0$; thus, variation of the action functional $\delta\chi_{\mathrm{MDR}} = 0$ produces a fully closed variational structure in the domain of matter dynamics.

Within this defined variational space, the theory is complete and consistent and possesses no open degrees of freedom outside the χ_{MDR} dynamics.

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NON-CIRCULARITY-DECLARATION:

The Lagrange-structure defined here is independent of any specific numerical

choice of \chi_{\mathrm{EPO}}(\varepsilon) or \alpha(\varepsilon).

All dynamical equations are derived solely within the variation space

\chi_{\mathrm{MDR}}; empirical quantities enter only later as boundary

or calibration conditions.

No observational relation is used simultaneously as input assumption and as

"prediction" of the same equation. Hence, the action framework is formally non-circular.

Formal Definitions and Closure Conditions of the \mathbf{\chi}_{\mathrm{MDR}}-Lagrange Structure

This appendix establishes all formal definitions, normalizations, and model limits of the ISOCH action framework. It serves to uniquely identify all quantities used in the text, their dynamics, coupling conditions, and normalized boundary values.

Empirical Derivation and Theoretical Embedding of the \mathbf{\chi}_{\mathrm{MDR}} Drift

The trend of the matter-dynamics rate \chi_{\mathrm{MDR}} is derived exclusively from mutually independent observational data sets. In the CMB phase-shift test (Planck 2018 data), the first empirical fixed-point determination of the \chi_{\mathrm{MDR}} drift is obtained from the phase position of the acoustic peaks n=1\mathrm{–6.

In a second, fully independent Lyman-α-BAO test (BOSS / eBOSS, z=2.3–2.55), the same drift behavior is found. Both data sets are physically independent and define the observationally based form of the epoch-dependent dynamics rate.

This empirically determined trend is not used as input to the Lagrangian variation, but exclusively for the calibration of the free parameters of the ISOCH potential $V\left(\chi_{\mathrm{MDR}},\varepsilon\right)$.

On this basis, all theoretical quantities become predictive, including the epoch-dependent energy density $\Omega_\chi\left(\varepsilon\right)$, the expansion history $H\left(\varepsilon\right)$, the local value of H_0, and residual effects in CMB and BAO measurements.

For every computation, every fit relation, and every derived formula, the following scheme is applied strictly once:

empirical $\mathbf{\chi}_{\mathrm{MDR}}$ drift $\rightarrow$ single potential calibration $\rightarrow$ theoretical prediction.

The multi-line test verifies the internal consistency of this method by demonstrating that different spectral lines and transitions within the same epoch window yield the same $\chi_{\mathrm{EPO}}\left(\varepsilon\right)$ normalization profile. The validation test then confirms that the once-calibrated potential $V\left(\chi_{\mathrm{MDR}},\varepsilon\right)$ reproduces all observed epoch relations without any further adjustment.

This excludes any multiple use of empirical relations or their appearance as ISOCH "predictions". The entire variational system is formally closed, empirically consistent, and logically non-circular.

Geometric Framework

The geometry follows the FLRW metric $g_{\mu\nu}=\mathrm{diag}\left(-,+,+,+\right)$ and is not varied.

$$\delta S_{\mathrm{geo}}=0.$$

An Einstein–Hilbert term is not used. The geometric background quantity $H\left(t\right)=\dot{a}/a$ is empirically determined and not a variational object. The Friedmann equation

$$H^2=\frac{8\pi G}{3}\rho_m$$

serves as the reference equation of GR, not as a consequence of the ISOCH action.

Dynamic and Non-Dynamic Quantities

Symbol Bedeutung Dynamisch

χ_{MDR}

Matter-dynamics rate (process-normalized scalar variable) ✅

$K_{\chi_{\mathrm{MDR}}}$

kinetic normalization factor, positive, normalized to H_0

$V\left(\chi_{\mathrm{MDR}},\varepsilon\right)$ epoch-dependent potential

ε epoch-coordinated normalization $\varepsilon=\ln{a}=-\ln{\left(1+z\right)}$

$H\left(t\right)$ empirical function of the FLRW background

$\mathcal{M}\left(\chi_{\mathrm{em}},\chi_{\mathrm{obs}}\right)$ observer–emitter normalization function

ρ_c

critical energy density $=3H_0^2/\left(8\pi G\right)$

$\frac{\partial V}{\partial\varepsilon}=0, \nabla_\mu T^{\mu\nu}=0.$

Lagrangian Density and Variational Principle

$\mathcal{L}_{\chi_{\mathrm{MDR}}}=\frac{1}{2}K_{\chi_{\mathrm{MDR}}}g^{\mu\nu}\nabla_\mu\chi_{\mathrm{MDR}}\nabla_\nu\chi_{\mathrm{MDR}}-V\left(\chi_{\mathrm{MDR}},\varepsilon\right).$

The variation with respect to χ_{MDR} yields:

$K_{\chi_{\mathrm{MDR}}}\nabla_\mu\nabla^\mu\chi_{\mathrm{MDR}}+\frac{\partial V}{\partial\chi_{\mathrm{MDR}}}=0.$

The geometry is assumed to be observation-fixed; no variation with respect to $g_{\mu\nu}$ takes place. Thus, ISOCH describes matter dynamics within the empirically defined FLRW background,

where $H\left(\varepsilon\right)$ is treated as a fixed background quantity. This approach ensures that the Lagrangian formalism acts exclusively on the matter component and avoids a double variation of the spacetime metric.

In FLRW:

${\ddot{\chi}}_{\mathrm{MDR}}+3H{\dot{\chi}}_{\mathrm{MDR}}+\frac{1}{K_{\chi_{\mathrm{MDR}}}}\frac{\partial V}{\partial\chi_{\mathrm{MDR}}}=0.$

Energy–Momentum Tensor and Continuity

$$T_{\mu\nu}=K_{\chi_{\mathrm{MDR}}}\partial_\mu\chi_{\mathrm{MDR}}\partial_\nu\chi_{\mathrm{MDR}}-g_{\mu\nu}\left[\frac{1}{2}K_{\chi_{\mathrm{MDR}}}\partial_\alpha\chi_{\mathrm{MDR}}\partial^\alpha\chi_{\mathrm{MDR}}-V\left(\chi_{\mathrm{MDR}},\varepsilon\right)\right].$$

$$\rho_{\chi_{\mathrm{MDR}}}=\frac{1}{2}K_{\chi_{\mathrm{MDR}}}\left({\dot{\chi}}_{\mathrm{MDR}}\right)^2+V,\qquad p_{\chi_{\mathrm{MDR}}}=\frac{1}{2}K_{\chi_{\mathrm{MDR}}}\left({\dot{\chi}}_{\mathrm{MDR}}\right)^2-V.$$

$${\dot{\rho}}_{\chi_{\mathrm{MDR}}}+3H\left(\rho_{\chi_{\mathrm{MDR}}}+p_{\chi_{\mathrm{MDR}}}\right)=0.$$

The condition $\nabla_\mu T^{\mu\nu}=0$ is satisfied. $T_{\mu\nu}$ is defined as the functional derivative of the matter part of the action, while the geometry remains externally fixed. The background fulfills $\delta S_{\mathrm{geo}}=0$ and $\nabla_\mu T^{\mu\nu}=0$ follows as a subsidiary condition. Thus, the definition of the energy–momentum tensor remains fully consistent with the fixed FLRW geometry.

Potential Forms and Normalization

Linear Potential:

$$V_{\mathrm{lin}}\left(\chi_{\mathrm{MDR}}\right)=\Lambda_{\chi_{\mathrm{MDR}}}^3\left(\chi_{\mathrm{MDR}}-1\right),\quad V_{\mathrm{lin}}\left(1\right)=0.$$

For the linear potential, the domain of definition is $\chi_{\mathrm{MDR}}\geq1$; thus $V_{\mathrm{lin}}\geq0$, and the WEC condition is globally satisfied. This restriction defines the admissible range of the matter-dynamics rate and excludes negative energy densities.

Quadratic Potential (Standard Case):

$$V_{\mathrm{quad}}\left(\chi_{\mathrm{MDR}}\right)=\frac{1}{2}m_{\chi_{\mathrm{MDR}}}^2\left(\chi_{\mathrm{MDR}}-1\right)^2,\quad V_{\mathrm{quad}}\left(1\right)=0.$$

The quadratic potential is the primary model case for all variational and dynamic derivations in ISOCH. It possesses a well-defined minimum at $\chi_{\mathrm{MDR}}=1$, and $m_{\mathrm{eff}}^2>0$ ensures the stability of the χ_{MDR}-Dynamik. The linear potential is not used as an independent variational potential but solely as a local effective approximation for $\left|\chi_{\mathrm{MDR}}-1\right|\ll1$ and for residual or sensitivity comparisons.

The epoch dependence of the potential appears exclusively through empirically calibrated parameters $\alpha\left(\varepsilon\right)$; explicit variation with respect to ε is omitted. Formally, therefore,

$V\left(\chi_{\mathrm{MDR}};\alpha\left(\varepsilon\right)\right)$ with $\frac{\partial V}{\partial \varepsilon}=0$.

This representation ensures the consistency of the variational principle without additional source terms.

Epoch Function and Redshift Relation

This relation serves solely to connect theoretical and observed quantities. It is not a result of the variation of the action but is defined as a measurement postulate. $\mathcal{M}$ thus functions as a bridge between model parameters and observational data.

$$\varepsilon=f\left(z\right)\equiv\ln{a}\left(z\right)=-\ln{\left(1+z\right)}.$$

$$1+z=\mathcal{M}\left(\chi_{\mathrm{em}},\chi_{\mathrm{obs}}\right),\qquad \mathcal{M}\left(\chi_{\mathrm{em}},\chi_{\mathrm{obs}}\right)=\frac{\chi_{\mathrm{obs}}}{\chi_{\mathrm{em}}}.$$

For $\chi_{\mathrm{MDR}}\rightarrow1$, it follows that $\mathcal{M}\rightarrow1$ and $z\rightarrow0$.

Linear Perturbations

$$\delta{\ddot{\chi}}_{\mathrm{MDR}}+3H\delta{\dot{\chi}}_{\mathrm{MDR}}+\left(\frac{k^2}{a^2}+m_{\mathrm{eff}}^2\right)\delta\chi_{\mathrm{MDR}}=0,$$
$$m_{\mathrm{eff}}^2=V^{\prime\prime}|_{\chi_{\mathrm{MDR}}}.$$

Solution in the Underdamped Regime:

$$\delta\chi_{\mathrm{MDR}}\propto e^{-3Ht/2}\sin{\left(\omega_kt+\phi\right)},\qquad \omega_k^2>\left(\frac{3H}{2}\right)^2.$$

Friedmann Closure

Since the geometry is not varied, the closure is carried out empirically via:

$$H\left(\varepsilon\right)=H_0\sqrt{\Omega_me^{-3\varepsilon}+\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon\right)}.$$

$\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon\right)$ is defined from the ISOCH energy density:

$\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon\right)=\frac{8\pi G}{3H_0^2}\left[\frac{1}{2}K_{\chi_{\mathrm{MDR}}}\left({\dot{\chi}}_{\mathrm{MDR}}\right)^2+V\left(\chi_{\mathrm{MDR}},\varepsilon\right)\right].$

With the calibrated parameters

$\frac{\Lambda_{\chi_{\mathrm{MDR}}}^3}{K_{\chi_{\mathrm{MDR}}}}\sim H_0^3$, und $\frac{m_{\chi_{\mathrm{MDR}}}^2}{K_{\chi_{\mathrm{MDR}}}}\sim H_0^2$.

the value for the present epoch $\varepsilon=\ 0$ is obtained as $\Omega_{\chi_{\mathrm{MDR}}}\left(0\right)\approx0.73$. This confirms the internal consistency of the normalization and allows direct comparability with the observed density parameters.

GR Limit

$\chi_{\mathrm{MDR}}\rightarrow1\Rightarrow{\dot{\chi}}_{\mathrm{MDR}}\rightarrow0,\;V\left(\chi_{\mathrm{MDR}},\varepsilon\right)\rightarrow0,\quad\Rightarrow\ \ \rho_{\chi_{\mathrm{MDR}}},p_{\chi_{\mathrm{MDR}}}\rightarrow0,$

$H^2\rightarrow\frac{8\pi G}{3}\rho_m.$

Normalization Factor and Dimensions

For unambiguous reproducibility, $K_{\chi_{\mathrm{MDR}}}$ is fixed to 1 in units of H_0^2. All specified parameters and sensitivities refer to this absolute normalization.

$K_{\chi_{\mathrm{MDR}}}=K_0>0,\ K_0\equiv1\mathrm{\ in\ units\ of\ }H_0^2.$

$\left[\chi_{\mathrm{MDR}}\right]=1,\left[K_{\chi_{\mathrm{MDR}}}\right]=1,\left[V\right]=\left[H^2\right],\left[\rho\right]=\left[H^2\right].$

Parametric Calibration

$\frac{\Lambda_{\chi_{\mathrm{MDR}}}^3}{K_{\chi_{\mathrm{MDR}}}}\sim H_0^3,\frac{m_{\chi_{\mathrm{MDR}}}^2}{K_{\chi_{\mathrm{MDR}}}}\sim H_0^2.$

The parameters are determined by minimization over the observed χ_{MDR} fluxes: This empirical fit serves exclusively to determine the numerical parameter combinations $\Lambda_{\chi_{\mathrm{MDR}}}^3/K_{\chi_{\mathrm{MDR}}}$ und

$m_{\chi_{\mathrm{MDR}}}^2/K_{\chi_{\mathrm{MDR}}}$ externally; it does not feed back into the Lagrangian variation and does not modify the action or the equation-of-motion system of the χ_{MDR} dynamics.

$$\min{\mathrm{param}}\sum_{i}\frac{\left(\chi_{\mathrm{obs},i}-\chi_{\mathrm{MDR,model},i}\right)^2}{\sigma_i^2}$$

Stability Conditions

$$K_{\chi_{\mathrm{MDR}}}>0,\quad m_{\mathrm{eff}}^2>0.$$

Thus, no tachyonic or ghost modes exist.

Energy Conservation, WEC Condition, and Closure Relations

The process-normalized matter-dynamics rate χ_{MDR} satisfies the weak energy condition (WEC) and local energy conservation. With positive, constant $K_{\chi_{\mathrm{MDR}}}$ and epoch-fixed $H(\varepsilon)$, the following holds:

$$\rho_{\chi_{\mathrm{MDR}}}+p_{\chi_{\mathrm{MDR}}}=K_{\chi_{\mathrm{MDR}}}\left({\dot{\chi}}_{\mathrm{MDR}}\right)^2\geq0,\rho_{\chi_{\mathrm{MDR}}}\geq0, V\left(\chi_{\mathrm{MDR}},\varepsilon\right)\geq0.$$

Local energy conservation follows directly from $\nabla_\mu T^{\mu\nu}=0$:

$${\dot{\rho}}_{\chi_{\mathrm{MDR}}}+3H\left(\varepsilon\right)\left(\rho_{\chi_{\mathrm{MDR}}}+p_{\chi_{\mathrm{MDR}}}\right)=0.$$

Thus, the weak energy condition is globally satisfied, and the dynamics are energetically and consistently closed.

For completeness of the model closure, the following also apply:

Geometry fixed (no Einstein–Hilbert term).

ε non-variable $\left(\partial V/\partial\varepsilon=0\right)$.

H empirically defined $\left(H=H\left(\varepsilon\right)\right)$.

Potentials normalized $\left(V\left(1\right)=0\right)$.

Linear perturbations in the underdamped regime.

$K_{\chi_{\mathrm{MDR}}}$ constant and positive.

Calibration via observed χ_{MDR} fluxes.

Energy conservation ensured.

GR limit reproduced.

Final Equation of Model Closure

The ISOCH theory is closed by:

$$\left(\chi_{\mathrm{MDR}},{\dot{\chi}}_{\mathrm{MDR}},V,K_{\chi_{\mathrm{MDR}}},H,\varepsilon\right)$$

with the following conditions:

$$\nabla_\mu T^{\mu\nu}=0,\qquad \partial V/\partial\varepsilon=0,\qquad K_{\chi_{\mathrm{MDR}}}=\mathrm{const.},\quad H=H\left(\varepsilon\right).$$

It contains no undefined quantities and no external variational parameters.

Supplementary Section – Clarifications on Variational Structure, Domain of Validity, and Empirical Coupling

This section resolves remaining definitional ambiguities in the formulation of the potential, the variational domain, the energy conditions, and the empirical H- closure. All clarifications serve to ensure the complete self-consistency of the ISOCH action framework.

Potential Structure and ${\varepsilon}$- Dependence

The potential is defined exclusively as a function of χ_{MDR}:

$$V=V(\chi_{\mathrm{MDR}}),$$

with ε serving as a non-variable epochal normalization. An ε -dependence can only be introduced indirectly through the calibration parameter fit, for example via empirical functions $V\left(\chi_{\mathrm{MDR}};\alpha\left(\varepsilon\right)\right)$. Accordingly, the consistent condition holds:

$$\frac{\partial V}{\partial\varepsilon}=0,\frac{\mathrm{d} V}{\mathrm{d\varepsilon}}=\frac{\partial V}{\partial\chi_{\mathrm{MDR}}}\frac{\mathrm{d} \chi_{\mathrm{MDR}}}{\mathrm{d\varepsilon}}.$$

This representation preserves the variational condition and fully resolves the apparent contradiction between "epoch-dependent" and "non-variable.

WEC Domain and Linear Potential

For the linear potential, the following holds:

$V_{\mathrm{lin}}\left(\chi_{\mathrm{MDR}}\right)=\Lambda_{\chi_{\mathrm{MDR}}}^3\left(\chi_{\mathrm{MDR}}-1\right), \qquad V_{\mathrm{lin}}\left(1\right)= 0.$

There exists an explicit domain of definition

$\chi_{\mathrm{MDR}}\geq1,$

which ensures $V_{\mathrm{lin}}\geq0$ and that the weak energy condition (WEC) is globally satisfied:

$\rho_{\chi_{\mathrm{MDR}}}+p_{\chi_{\mathrm{MDR}}}=K_{\chi_{\mathrm{MDR}}}\left({\dot{\chi}}_{\mathrm{MDR}}\right)^2\geq0.$

For global normalization, the offset-normalized form can equivalently be used:

$V_{\mathrm{lin,norm}}\left(\chi_{\mathrm{MDR}}\right)=\Lambda_{\chi_{\mathrm{MDR}}}^3\left|\chi_{\mathrm{MDR}}-1\right|.$

This representation preserves $V\geq0$ over the entire domain of definition without altering the behavior of the equation of motion or the stability conditions. Thus, the WEC domain is uniquely determined, and the energy positivity of the ISOCH potential is globally ensured.

The validity range of the linear potential

$V_{\mathrm{lin}}\left(\chi_{\mathrm{MDR}}\right)=\Lambda_{\chi_{\mathrm{MDR}}}^3\left(\chi_{\mathrm{MDR}}-1\right)$

is restricted to the domain $\chi_{\mathrm{MDR}}\geq1$. It is used exclusively as an effective approximation in the immediate vicinity of $\chi_{\mathrm{MDR}}\approx1$. The standard evolution and all variational proofs of the ISOCH model are based on the quadratic potential V_{quad}.

Empirical H-Closure and Algorithmic Corollary

The Hubble function $H(\varepsilon)$ is not a variational object but an empirically defined input term of the background. It is not derived from the variation of the action but is set based on observational constraints and serves as a constraint for fixing the geometry.

$$H\left(\varepsilon\right)=H_0\sqrt{\Omega_me^{-3\varepsilon}+\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon\right)}.$$

The ISOCH energy density of the matter-dynamics rate is given by:

$$\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon\right)=\frac{8\pi G}{3H_0^2}\left[\frac{1}{2}K_{\chi_{\mathrm{MDR}}}\left({\dot{\chi}}_{\mathrm{MDR}}\right)^2+V\left(\chi_{\mathrm{MDR}}\right)\right].$$

For numerical determination, $H(\varepsilon)$ is computed iteratively. Starting from $H^{\left(0\right)}=H_0$, the following applies:

$$H^{\left(n+1\right)}=H_0\sqrt{\Omega_me^{-3\varepsilon}+\Omega_{\chi_{\mathrm{MDR}}}^{\left(n\right)}};$$

The computational step proceeds algorithmically:

compute $\Omega_{\chi_{\mathrm{MDR}}}^{\left(n\right)}$ from $\chi_{\mathrm{MDR}}^{\left(n\right)}$; update $H^{\left(n+1\right)}$ according to the above relation.

The relation is contractive in the range $H/H_0 \in [0.5,1.5]$. Since $\frac{\partial H^{\left(n+1\right)}}{\partial H^{\left(n\right)}}<1$ is satisfied for all physically relevant parameters, the sequence $H^{\left(n\right)}$ converges to a unique fixed point $H(\varepsilon)$.

Thus, the existence and uniqueness of the empirical closure are formally guaranteed.

For formal clarification of the empirical–variational separation, the following applies:

The ISOCH theory treats the Hubble function $H\left(\varepsilon\right)$ in two logically separated stages to ensure self-consistency between the empirically calibrated geometry and the energy density $\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon\right)$ defined by χ_{MDR}.

In the first stage, $H\left(\varepsilon\right)$ is treated as an observation-based fixed background quantity. During variation,

$$\delta g_{\mu\nu}=0,\delta H\left(\varepsilon\right)=0,$$

so that the action variation operates exclusively within the domain of matter dynamics χ_{MDR}. $H\left(\varepsilon\right)$ acts only as an exogenous damping term in the equations of motion.

In the second stage, after the variation, it is verified whether the energy density $\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon\right)$ yields a closure relation

$$H^2\left(\varepsilon\right)=H_0^2\left[\Omega_m\mathrm{e}^{\mathrm{-3\varepsilon}}+\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon\right)\right]$$

consistent with the empirical observations of $H\left(\varepsilon\right)$. The fixed-point iteration thereby constitutes no additional variation but a subsequent consistency check between theoretical energy density and empirical geometry.

This formulation removes any apparent tension between the "exogenous" and "endogenous" treatments of $H\left(\varepsilon\right)$; the variation remains strictly confined to χ_{MDR}, while the geometry retains its empirically fixed character within the ISOCH theory.

The separation between the dynamic matter-dynamics rate χ_{MDR} and the fixed geometry $H(\varepsilon)$ remains unambiguous. The fixed-point mapping ensures a stable and deterministic closure of the ISOCH theory.

Convergence of the H-Closure:

The iterative relation constitutes a contractive mapping in the range $H/H_0\in\left[0.5,1.5\right]$. Since $\partial H^{\left(n+1\right)}/\partial H^{\left(n\right)}<1$ is satisfied for all physically relevant parameters, the sequence $H^{\left(n\right)}$ converges toward a unique fixed point $H\left(\varepsilon\right)$. Thus, the existence and uniqueness of the closure are mathematically guaranteed, even though H is not derived directly from variation but determined from observational constraints; the resulting fixed-point mapping nonetheless ensures a unique, convergent, and stable closure within the ISOCH regime.

This empirical closure does not couple the geometry dynamically to the action but serves as an observational constraint. Derivation of the Friedmann formula from the ISOCH action is not required; H represents a fixed background quantity. Thus, the separation between the dynamic matter rate χ_{MDR} and the fixed geometry $H(\varepsilon)$ is unambiguous.

Initial Conditions of the ${\chi}_{\mathrm{MDR}}$ Dynamics

For the integration of the equation of motion, defined initial values are set:

$$\chi_{\mathrm{MDR}}\left(\varepsilon=0\right)=1,{\dot{\chi}}_{\mathrm{MDR}}\left(\varepsilon=0\right)=0.$$

These initial conditions fix the normalization of the matter-dynamics rate χ_{MDR} to the present epoch $\varepsilon\ =\ 0$ and ensure that $V\left(1\right)=0$ and $\rho_{\chi_{\mathrm{MDR}}},p_{\chi_{\mathrm{MDR}}}\rightarrow0$ in the GR limit.

Numerical Calibration and Reproducibility

The parameter combinations

$$\frac{\Lambda_{\chi_{\mathrm{MDR}}}^3}{K_{\chi_{\mathrm{MDR}}}}\sim H_0^3,\frac{m_{\chi_{\mathrm{MDR}}}^2}{K_{\chi_{\mathrm{MDR}}}}\sim H_0^2.$$

are calibrated through empirical fits to Ω_{m} and $\Omega_{\chi_{\mathrm{MDR}}}\left(\varepsilon=0\right)\approx0.73$. The calibration is performed by minimizing

$$\min{\mathrm{param}}\sum_{i}\frac{\left(\chi_{\mathrm{obs},i}-\chi_{\mathrm{MDR,model},i}\right)^2}{\sigma_i^2}$$

where H_0 and Ω_m are fixed, and χ_{MDR} is adjusted parametrically. This definition makes the ISOCH parameters fully reproducible.

Redshift Relation

The defined observer–emitter relation

$$1+z=\mathcal{M}\left(\chi_{\mathrm{em}},\chi_{\mathrm{obs}}\right)=\frac{\chi_{\mathrm{obs}}}{\chi_{\mathrm{em}}}$$

is postulated by the ISOCH normalization. It is not derived from the action but constitutes the epoch-dependent correspondence $\varepsilon\ \leftrightarrow\ z$. This uniquely establishes the observational normalization without introducing additional degrees of freedom.

Potential Definition (Consistency Theorem)

$$\mathrm{We\ use\ }V=V\left(\chi_{\mathrm{MDR}}\right)\mathrm{\ with\ }\frac{\partial V}{\partial\varepsilon}=0.$$

Epoch information enters exclusively through the parameterization $\alpha\left(\varepsilon\right)$ or via the fit $V\left(\chi_{\mathrm{MDR}};\alpha\left(\varepsilon\right)\right)$, not through an explicit ε-variation. Thus, $\nabla_\mu T^{\mu\nu}=0$ always holds, and no additional source-term coupling arises.

Variable Conventions (Dual Epoch Notation)

The theoretical formulation uses $\varepsilon=\ln{a}=-\ln{\left(1+z\right)}$ as the internal coordinate and z as the empirical observational variable, connected by the bridging function

$$\varepsilon=f\left(z\right)=-\ln{\left(1+z\right)}.$$

Thus, the notation is doubly defined but unambiguously linked; confusion between theoretical and observational normalization is excluded.

Fixed-Point Clarification “H Exogenous”

$H\left(\varepsilon\right)$ is not a variational object but an exogenous input term of the background. All expressions in the action use $H\left(\varepsilon\right)$ as a predetermined function; consequently, the Lagrangian variation is restricted solely to χ_{MDR}. This corresponds to the ISOCH definition of a fixed geometry without an Einstein–Hilbert term.

Convergence and Uniqueness of the H-Closure

The iterative relation

$$H^{\left(n+1\right)}=H_0\sqrt{\Omega_me^{-3\varepsilon}+\Omega_{\chi_{\mathrm{MDR}}}^{\left(n\right)}\left(\varepsilon\right)}$$

constitutes a contractive mapping in the interval $H/H_0\in\left[0.5,1.5\right]$. Since $\partial H^{\left(n+1\right)}/\partial H^{\left(n\right)}< 1$ holds for all physically relevant parameters, the sequence $H^{\left(n\right)}$ converges to a unique fixed point $H\left(\varepsilon\right)$. Thus, the existence and uniqueness of the closure are formally guaranteed.

Formal Convergence of the Empirical H-Closure

To ensure mathematical completeness, the fixed-point iteration used in the ISOCH formulation,

$$H_{n+1}=H_0\sqrt{\Omega_me^{-3\varepsilon}+\Omega_{\chi_{\mathrm{MDR}}}\left(H_n\right)}$$

is formally demonstrated to be a contractive mapping.

The energy-density component is given by

$$\Omega_{\chi_{\mathrm{MDR}}}\left(H\right)=\frac{8\pi G}{3H_0^2}\left(\frac{1}{2}K_{\chi_{\mathrm{MDR}}}H^2\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2+V\left(\chi_{\mathrm{MDR}}\right)\right), d\varepsilon=H dt.$$

Differentiation yields

$$F^\prime\left(H\right)=\frac{H_0^2}{\sqrt{\Omega_me^{-3\varepsilon}+\Omega_{\chi_{\mathrm{MDR}}}\left(H\right)}}\frac{d\Omega_{\chi_{\mathrm{MDR}}}}{dH}.$$

For the explicit part, the following holds

$$\left|\frac{d\Omega_{\chi_{\mathrm{MDR}}}}{dH}\right|\le A K_{\chi_{\mathrm{MDR}}}\left|H\right|\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2, A=\frac{8\pi G}{3H_0^2}.$$

At the fixed point $H=H^\ast$, it follows

$$\left|F^\prime\left(H^\ast\right)\right|\le\frac{4\pi G}{3} K_{\chi_{\mathrm{MDR}}}\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2.$$

With the empirically calibrated values

$$\left|\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right|\lesssim0.04,\quad K_{\chi_{\mathrm{MDR}}}=1,\frac{8\pi G}{3}=1,$$

one obtains

$$\left|F^\prime\left(H^\ast\right)\right|\lesssim8\times{10}^{-4}\ll1.$$

According to the Banach fixed-point theorem, the mapping is strictly contractive; the iteration converges uniquely and independently of the initial value toward the fixed point $H\left(\varepsilon\right)$. Thus, the existence and uniqueness of the empirical H- closure are mathematically ensured.

Note: Relation (1) simultaneously defines a Lipschitz constant

$$L\le8\times{10}^{-4},$$

and all numerical integrations satisfy $L<1$ in the range

$$H/H_0\in\left[0.5,1.5\right].$$

Hence, the numerical convergence of the empirically calibrated iteration is fully characterized. The subsequent derivation shows that this stability results from the general regularity conditions of the potential.

The ISOCH theory is therefore not only physically but also formally and analytically fully closed.

Lemma 1 – Contractive Property of the H-Iteration (Formal Proof)

For the ISOCH closure scheme

$$H_{n+1}=F\left(H_n\right)=H_0\sqrt{\Omega_me^{-3\varepsilon}+\Omega_{\chi_{\mathrm{MDR}}}\left(H_n\right)}$$

the following assumptions hold:

The potential $V\left(\chi_{\mathrm{MDR}}\right)$ is continuously differentiable and possesses a finite Lipschitz constant $L_V>0$:

$$\left|V^\prime\left(\chi_{\mathrm{MDR},1}\right)-V^\prime\left(\chi_{\mathrm{MDR},2}\right)\right|\le L_V\left|\chi_{\mathrm{MDR},1}-\chi_{\mathrm{MDR},2}\right|.$$

The effective mass parameter lies above a positive lower bound $m_{\mathrm{eff}}^2\geq m_0^2>0$.

The deviation $\left|\chi_{\mathrm{MDR}}-1\right|$ remains within a small, bounded region with $\delta\ll1$.

The ratio H/H_0 lies within the empirically relevant interval $\left[0.5,1.5\right]$.

It then follows for the derivative of the iteration operator that:

$$\left|F^\prime\left(H\right)\right|\le C L_V\delta<1,$$

where C is a dimensionless constant of order unity, resulting from the normalization of the energy and damping terms. Hence, F acts contractively on the interval $\left[0.5,1.5\right]$, and there exists exactly one stable fixed point

$$H^\ast=F\left(H^\ast\right).$$

This fixed point defines the unique self-consistency of the ISOCH closure for all potential forms V, that satisfy the stated regularity conditions.

The subsequent empirical demonstration confirms the numerical realization of the contraction theorem formally established here.

Extended Stability and Convergence Assurance under Implicit $\mathbf{\chi}_{\mathrm{MDR}}\left({H}\right)$ Coupling

The practical iteration employs the fixed-point mapping

$$H_{n+1}=H_0\sqrt{\Omega_me^{-3\varepsilon}+\Omega_{\chi_{\mathrm{MDR}}}\left(H_n\right)}.$$

Up to this point, the convergence proof has been conducted under the assumption that χ_{MDR} is already evaluated at each iteration step. To ensure stability also in the implicitly coupled case $\chi_{\mathrm{MDR}}=\chi_{\mathrm{MDR}}\left(H,\varepsilon\right)$—that is, during simultaneous determination of χ_{MDR} and H within a coupled integration routine—the contribution of the implicit derivative $\partial\chi_{\mathrm{MDR}}^\prime/\partial H$ is taken into account.

With

$$\Omega_{\chi_{\mathrm{MDR}}}\left(H\right)=\frac{8\pi G}{3H_0^2}\left(\frac{1}{2}K_{\chi_{\mathrm{MDR}}}H^2\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2+V\left(\chi_{\mathrm{MDR}}\right)\right), d\varepsilon=H dt.$$

the total derivative yields

$$\frac{d\Omega_{\chi_{\mathrm{MDR}}}}{dH}=\frac{\partial\Omega_{\chi_{\mathrm{MDR}}}}{\partial H}+\frac{\partial\Omega_{\chi_{\mathrm{MDR}}}}{\partial\chi_{\mathrm{MDR}}^\prime}\frac{\partial\chi_{\mathrm{MDR}}^\prime}{\partial H}.$$

The explicit term provides, as in the basic proof, the bound

$$\left|F^\prime\left(H\right)\right|\le L_0=\frac{4\pi G}{3} K_{\chi_{\mathrm{MDR}}}\left(\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right)^2\ll1.$$

The backreaction of the χ_{MDR} dynamics on the H-iteration follows from the homogeneous equation of motion

$$K_{\chi_{\mathrm{MDR}}}\left(\chi_{\mathrm{MDR}}^{\prime\prime}+3\chi_{\mathrm{MDR}}^\prime+H^{-2}\frac{\partial V}{\partial\chi_{\mathrm{MDR}}}\right)=0,$$

whose partial differentiation with respect to

$$K_{\chi_{\mathrm{MDR}}}\left(\frac{\partial\chi_{\mathrm{MDR}}^{\prime\prime}}{\partial H}+3\frac{\partial\chi_{\mathrm{MDR}}^\prime}{\partial H}\right)+\frac{\partial}{\partial H}\left(H^{-2}\frac{\partial V}{\partial\chi_{\mathrm{MDR}}}\right)=0.$$

For $u\left(\varepsilon\right)=\partial\chi_{\mathrm{MDR}}^\prime/\partial H$, a Grönwall estimate gives

$$\left|\frac{\partial\chi_{\mathrm{MDR}}^\prime}{\partial H}\right|\le C_\chi e^{-\int3 d\varepsilon}\le C_\chi e^{-3\varepsilon},$$

where C_χ is finite (e.g., from $\chi_{\mathrm{MDR}}^\prime\left(0\right)=0$. Thus, $\left|\partial\chi_{\mathrm{MDR}}^\prime/\partial H\right|$ remains bounded for all ε and decreases monotonically.

Substituting (Energy–Momentum Tensor) into (Euler–Lagrange equation) yields an effective Lipschitz constant

$$L_{\mathrm{eff}}=\left|F^\prime\left(H\right)\right|\le L_0+L_\chi, L_\chi=\frac{4\pi G}{3} K_{\chi_{\mathrm{MDR}}}\left|\frac{d\chi_{\mathrm{MDR}}}{d\varepsilon}\right|\left|\frac{\partial \chi_{\mathrm{MDR}}^\prime}{\partial H}\right|.\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \left(5\right)$$

With $\left|d\chi_{\mathrm{MDR}}/d\varepsilon\right|\lesssim0.04$ and $\left|\partial\chi_{\mathrm{MDR}}^\prime/\partial H\right|\lesssim{10}^{-2}$, it follows that $L_\chi\lesssim{10}^{-6}$, and thus

$$L_{\mathrm{eff}}\approx L_0\left(1+{10}^{-3}\right)\ll1.$$

The contraction condition $\left|F^\prime\left(H\right)\right|<1$ therefore also holds for the implicit coupling $\chi_{\mathrm{MDR}}\left(H,\varepsilon\right)$. The iterative H-closure thus converges uniquely and stably in both the explicit and implicitly coupled cases,

$$L_{\mathrm{eff}}<1\forall\varepsilon\in R.$$

Hence, the stability of the ISOCH fixed-point procedure is formally proven even for simultaneously solved χ_{MDR}-H couplings; the fixed-point procedure remains contractive, unique, and mathematically closed.

Variational Framework and Model Completeness

The ISOCH theory is defined as a partially variational theory. The variation of the action is performed exclusively within the space of the dynamic matter variable χ_{MDR}, while the geometric background function $H\left(\varepsilon\right)$ and the metric $g_{\mu\nu}$ are treated as empirically calibrated external constraints.

Thus, the framework does not represent an incomplete variational approach but a self-contained variational system within the χ_{MDR} subspace. The fixation of geometry corresponds to a fixed-background formalism: the spacetime metric is not varied but constitutes an observationally defined boundary condition of the partially variational action framework.

This procedure replaces spacetime variation with an internal process variation of matter dynamics $\chi_{\mathrm{MDR}}\left(\varepsilon\right)$ and leads to a mathematically closed set of independent Euler–Lagrange equations.

Formally, $\delta g_{\mu\nu}=0$ and $\delta H\left(\varepsilon\right)=0$; thus, variation of the action functional $\delta\chi_{\mathrm{MDR}}=0$ produces a fully closed variational structure in the domain of matter dynamics.

Within this defined variational space, the theory is complete and consistent and possesses no open degrees of freedom outside the χ_{MDR} dynamics.

[END MACHINE-READABLE FORMULAS-PRIMARY SOURCE]